

Ground-state energies and widths of ${}^5\text{He}$ and ${}^5\text{Li}$ nuclei

V.D. Efros* and H. Oberhummer

Institut für Kernphysik, Wiedner Hauptstr. 8–10, TU Wien, A-1040 Vienna, Austria
(February 9, 2008)

We extract energies and widths of the ground states of ${}^5\text{He}$ and ${}^5\text{Li}$ from recent single-level R-matrix fits to the spectra of the ${}^3\text{H}(\text{d}, \gamma){}^5\text{He}$ and the ${}^3\text{He}(\text{d}, \gamma){}^5\text{Li}$ reactions. The widths obtained differ significantly from the formal R-matrix values but they are close to those measured as full widths at half maxima of the spectra in various experiments. The energies are somewhat lower than those given by usual estimates of the peak positions. The extracted values are close to the S-matrix poles calculated previously from the multi-term analyses of the N- ${}^4\text{He}$ elastic scattering data.

PACS numbers: 25.40.Hs, 25.40.Ny

The ground-state properties of ${}^5\text{He}$ and ${}^5\text{Li}$ nuclei were studied in several experiments, see Refs. [1–3] for a review. In the most recent work [3] the values of the $A = 5$ widths were reported which are significantly larger than the previous ones. In the present work we reconsider the experimental data of Ref. [3]. We extract the widths and the positions of the $A=5$ levels from these data.

The present work has been done in connection with some applications of an astrophysical interest [4–6]. Besides, one may note that at present there exist difficulties in obtaining p-wave N- ${}^4\text{He}$ phase shifts at low-energy from a realistic non-central NN interaction (see e.g. [7]) which hinders a microscopic description of light multicluster nuclear systems (see e.g. [8]). The phase shifts are governed by the ${}^5\text{He}$ and ${}^5\text{Li}$ ground-state resonances and it would be convenient to analyze the problem directly in terms of their energies and widths.

In Ref. [3] a single-term R-matrix fit to experimental γ -ray spectra of that work was undertaken to determine the widths, while in most of the previous work the widths were estimated by measuring FWHM of the spectra (see Table 1 in Ref. [3]). The ground-state energies were mainly estimated from the positions of the peaks (see [2]). In Ref. [9] the ${}^5\text{He}$ energy and width were extracted from an R-matrix fit to the whole set of the ${}^4\text{He}(\text{n}, \text{n}){}^4\text{He}$ cross section and polarization data in the elastic region. The p-wave R-matrix elements were approximated by resonant terms plus linear functions in energy representing contributions from distant R-matrix levels. The position of the pole of the expression obtained was found giving the energy and width of ${}^5\text{He}$. In Ref. [10] a similar type analysis was carried out for ${}^4\text{He}(\text{n}, \text{n}){}^4\text{He}$ and ${}^4\text{He}(\text{p}, \text{p}){}^4\text{He}$ scattering using another form of the S-matrix parameterization. These multi-term analyses are considerably influenced by potential scattering interference. It is of interest to study the pole structure of the S-matrix for other types of processes involving $A = 5$ nuclei.

In Ref. [3] the ${}^3\text{H}(\text{d}, \gamma){}^5\text{He}$ and the ${}^3\text{He}(\text{d}, \gamma){}^5\text{Li}$ spectra have been measured at $E_{\text{d}} = 8.6 \text{ MeV}$ and $\theta_{\text{lab}} = 90^\circ$. The spectra were fitted by the expression¹

$$\frac{\Gamma_R(E_{N\alpha})}{[E_{N\alpha} - E_{\text{res}} - \Delta(E_{N\alpha})]^2 + \left[\frac{\Gamma_R(E_{N\alpha})}{2}\right]^2} \quad (1)$$

times a slowly varying function in the energy $E_{N\alpha}$. Here $E_{N\alpha}$ is the relative motion energy of the $(N+\alpha)$ subsystem, and $\Delta(E_{\text{res}}) = 0$. The energy dependencies of the width $\Gamma_R(E)$ and of the shift function $\Delta(E)$ corresponded to the single-level R-matrix expression, that is $\Gamma_R(E) = 2\gamma^2 P(E)$ and $\Delta(E) = -\gamma^2[S(E) - S(E_{\text{res}})]$. Here γ^2 is the reduced width, while the penetrability P and the shift function S for the $\ell = 1$ orbital momentum are defined as usual [11]. The overall normalizations of the spectra and the reduced widths were fitted to the experimental data while for the E_{res} energies the values from Ref. [1] were used. As a result the following values of the widths $\Gamma_R(E_{\text{res}})$ were obtained which we shall denote Γ_R

$$\Gamma_R({}^5\text{He}) = (1.36 \pm 0.19) \text{ MeV}, \quad \Gamma_R({}^5\text{Li}) = (2.44 \pm 0.21) \text{ MeV}. \quad (2)$$

Here and below the input parameters are [3]: the channel radius $R = 3.6 \text{ fm}$, $E_{\text{res}} = 0.89 \text{ MeV}$, $\gamma^2 = (3.32 \pm 0.46) \text{ MeV}$ in the ${}^5\text{He}$ case, and $R = 3.6 \text{ fm}$, $E_{\text{res}} = 1.97 \text{ MeV}$, $\gamma^2 = (3.33 \pm 0.29) \text{ MeV}$ in the ${}^5\text{Li}$ case. In Eq. (2) and below the errors correspond to those in the reduced widths.

*Permanent address: Russian Research Centre "Kurchatov Institute", 123182 Moscow, Russia

¹In Eq. (1) we express the spectra in terms of the energy $E_{N\alpha}$ that coincides with E_N from Ref. [3]. Our E_{res} coincides with that of Ref. [3] in their Table II but differs from E_{res} in their Eq. (1). Our quantity $\Gamma_R(E)$ coincides with $\Gamma_N(E)$ from Ref. [3].

The previous values of the width ranged from (0.55 ± 0.03) MeV to (0.85 ± 0.05) MeV in the ${}^5\text{He}$ case. In the ${}^5\text{Li}$ case the previous widths were grouped around 1.5 MeV except for one value of (2.6 ± 0.4) MeV. The latter value [12] was obtained using the method in principle similar to that of Ref. [3], while the others were obtained in several experiments by measuring FWHMs of spectra for various reactions (see [1,3]) and with the help of the above-mentioned analyses [9,10] of the elastic $\text{n-}{}^4\text{He}$ scattering. The FWHM for the spectra of Ref. [3] are rather close to the above-mentioned values.

We shall extract the poles $E = E_0 - i\Gamma/2$ of the scattering matrix from the results of Ref. [3]. These poles determine the *physical* resonance energies E_0 and widths Γ that are different from E_{res} and $\Gamma_R = \Gamma_R(E_{\text{res}})$ entering Eq. (1). The difference proves sometimes to be quite significant, see e.g. [13]. In particular, namely Γ and not Γ_R determines the lifetime of a system as it is required for example in the applications [4–6]. One can see that $\Gamma_R(E_{\text{res}})$ cannot serve as the estimate of a width even in a narrow resonance case. The quantity $\Gamma_S(E) = [1 - \Delta'(E_{\text{res}})]^{-1}\Gamma_R(E)$ at $E = E_{\text{res}}$ could have served as such an estimate [11]. Here Δ' is a derivative over E taken at the point E_{res} . (The quantity $\Gamma_S(E)$ enters the resonant factor of a reaction when written in the form $\Gamma_S(E)/[(E - E_{\text{res}})^2 + (\Gamma_S(E)/2)^2]$.) In our case the widths are broad and the above-mentioned estimate is not very accurate, however.

In case of the parameterization of Eq. (1) the complex energy E of the resonance is the solution to the equation

$$[E - E_{\text{res}} - \Delta(E)]^2 + [\Gamma_R(E)/2]^2 = 0, \quad (3)$$

at the condition $k = (2\mu E)^{1/2}/\hbar = k_1 - ik_2$, $k_1 > 0$, $k_2 > 0$. In the ${}^5\text{He}$ case this equation can be represented in the form

$$(x - x_{\text{res}})^2[\gamma^{-2}\bar{E} \cdot (x + 1) + (x_{\text{res}} + 1)^{-1}]^2 + x^3 = 0,$$

where we introduced the notation $\bar{E} = \hbar^2/(2\mu R^2)$, μ being the reduced mass, and $x = E/\bar{E}$, $x_{\text{res}} = E_{\text{res}}/\bar{E}$. We obtained²

$$E_0({}^5\text{He}) = (0.735 \pm 0.02) \text{ MeV}, \quad \Gamma({}^5\text{He}) = (0.57 \pm 0.02) \text{ MeV}. \quad (4)$$

Note that the value of E_0 obtained is slightly shifted downwards with respect to the position of the peak of the spectrum which position equals to 0.81 MeV. This is due to an asymmetric form of the spectrum.

The values (4) should be somewhat corrected due to the following reason. The value of 0.89 MeV taken in Ref. [3] as E_{res} was actually obtained [2] as the mean position of the resonant peak. In general, this position differs from the optimal E_{res} value or, which is the same, from the energy at which the resonant phase shift reaches 90° . We changed the value of E_{res} to shift the position of the peak to the value of 0.89 MeV. The resulting E_{res} value, $E_{\text{res}} = 0.98$ MeV, is close to those given in Refs. [1,9,14] for the elastic $\text{n-}{}^4\text{He}$ scattering. It is not clear whether the small change in the E_{res} value made is quite compatible with the γ^2 value deduced in Ref. [3] from the fit to the data. The point is that in order to allow for spectrum calibration errors the fitting procedure in Ref. [3] included shifts up or down in energy of the entire observed spectrum resulting from a convolution with a detector response function. The information on the latter function listed in the paper [3] does not suffice to perform the convolution. If, nevertheless, one adopts the shifted E_{res} value one obtains the following energy and width of ${}^5\text{He}$:

$$E_0({}^5\text{He}) = (0.80 \pm 0.02) \text{ MeV}, \quad \Gamma({}^5\text{He}) = (0.65 \pm 0.02) \text{ MeV}. \quad (5)$$

These values are in a remarkable agreement with the values of $E_0 = 0.77$ MeV and $\Gamma = 0.64$ MeV, and also with the values of $E_0 = 0.78$ MeV and $\Gamma = 0.72$ MeV obtained in Refs. [9] and [10], respectively, from multi-term analyses of the elastic $\text{n-}{}^4\text{He}$ scattering. The measured FWHM (see Table 1 in Ref. [3]) are also in accordance with the width value obtained.

In the ${}^5\text{Li}$ case, $\Delta(E)$ and $\Gamma_R(E)$ entering Eq.(3) include Coulomb functions with complex arguments and their derivatives. They were calculated with the help of the computer code of Ref. [15]. Eq. (3) was solved with a Newton-type method. The results corresponding to Eqs. (4) and (5) are

$$E_0({}^5\text{Li}) = (1.635 \pm 0.03) \text{ MeV}, \quad \Gamma({}^5\text{Li}) = (1.16 \pm 0.03) \text{ MeV}, \quad (6)$$

²A pleasant peculiarity of the result of Eq. (4) for the width is a considerable reduction of the relative errors as compared to Eq. (2). The same holds true for Eqs. (5)–(7) below and this is due to the difference in the γ^2 -dependence in the equations determining the widths. Note as well that the highest (lowest) E_0 value, of course, corresponds to the lowest (highest) Γ value in Eq. (4) and in the relations below.

and

$$E_0(^5\text{Li}) = (1.72 \pm 0.03) \text{ MeV}, \quad \Gamma(^5\text{Li}) = (1.28 \pm 0.03) \text{ MeV}, \quad (7)$$

respectively. The E_0 values are in accordance with the value of $E_0 = 1.6$ MeV obtained in Ref. [10] from the multi-term analysis of the elastic $p\text{--}^4\text{He}$ scattering. Similar to the ^5He case these values are shifted downwards with respect to the peak position of 1.97 MeV. The widths obtained are lower than the value of $\Gamma = 1.45$ MeV from Ref. [10] and than the mean FWHM value [1] of ≈ 1.5 MeV, but they are close to the FWHM value of (1.24 ± 0.03) MeV for the spectrum of Ref. [3].

It may be noted that the above S -matrix pole widths cannot be "reduced" as were the R -matrix widths in Ref. [3] to obtain reduced widths γ^2 . The reduced widths of ^5He and ^5Li extracted in Ref. [3] turned out to be equal which showed that charge symmetry is not violated. The ratio of the R -matrix widths [3] thus has a direct physical meaning as well as the S -matrix pole results.

In conclusion, the experimental data of Ref. [3] on the $^3\text{H}(d, \gamma)^5\text{He}$ and the $^3\text{He}(d, \gamma)^5\text{Li}$ reactions allowed safe extrapolations to the S -matrix poles. From these extrapolations rather accurate values of the complex energies of $A = 5$ nuclei have been determined. They are in a good agreement with those obtained from the multi-term analyses of $N\text{--}^4\text{He}$ cross section and polarization data set in the elastic region. In spite of the broad resonances the widths obtained proved to be close to those measured as FWHM. However, the real parts of the poles are shifted downwards with respect to the positions of the peaks in the spectra.

This work was supported by Fonds zur Förderung der wissenschaftlichen Forschung in Österreich and by the International Science Foundation and Russian Government (grant J4M100).

- [1] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **A227**, 1 (1974); F. Ajzenberg-Selove, Nucl. Phys. **A320**, 1 (1979); **A413**, 1 (1984); **A490**, 1 (1988).
- [2] T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. **78**, 1 (1966).
- [3] M.J. Balbes, G. Feldman, L.H. Kramer, H.R. Weller, and D.R. Tilley, Phys. Rev. C **43**, 343 (1991).
- [4] J. Görres, H. Herndl, I.J. Thompson, and M. Wiescher, Phys. Rev. C **52**, 2231 (1995).
- [5] H. Oberhummer, W. Balogh, V.D. Efros, H. Herndl, and R. Hofinger, Few-Body Sys. Suppl., **8**, 317 (1995).
- [6] V.D. Efros, W. Balogh, H. Herndl, R. Hofinger, and H. Oberhummer, Z. Physik A, 1996, in press.
- [7] M.N. Ustinin and V.D. Éfros, Yad. Fiz. **49**, 1297 (1989) [Sov. J. Nucl. Phys. **49**, 807 (1989)].
- [8] A. Csótó, Phys. Rev. C **48**, 165 (1993).
- [9] J.E. Bond and F.W.K. Kirk, Nucl. Phys. **A287**, 317 (1977).
- [10] L. Kraus and L. Linck, Nucl. Phys. **A224**, 45 (1974).
- [11] A.M. Lane and R.G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
- [12] W. Buss, W. Del Bianco, H. Waffler, and B. Ziegler, Nucl. Phys. **A112**, 47 (1968).
- [13] G.M. Hale, R.E. Brown and N. Jarmie, Phys. Rev. Lett. **59**, 763 (1987).
- [14] Th. Stambach and R.L. Walter, Nucl. Phys. **A180**, 225 (1972).
- [15] I.J. Thompson and A.R. Barnett, Comput. Phys. Comm. **36**, 363 (1985).